

LCM & HCF by prime factorisation: answers

1 HCF LCM

a	2	24
b	1	65
c	3	60
d	7	105

2 HCF LCM

a	36	216
b	18	270
c	15	630
d	5	4725

3 HCF LCM

a	18	756
b	6	792
c	25	4875
d	63	8316

4 HCF LCM

a	6	720
b	6	420
c	7	784

5 646 800

6 9:36 am

7 6 pupils per team

8 3:20 pm

9 2025

10 10 times

11a 3 packets of cheese,

5 boxes of burgers

11b 60 bread rolls

12 9 and 15, or 3 and 45

13 $x = 60$

14 25, 50, 75, 150 s

15 $8x^4y^3z$

16 $84a^4b^5c^6$

17 500

18 $x = 8$

19 98^{91}

20 **B** 105

N $Q + R + S = 31$

How every question is done

- 1 Write each number as a product of prime factors, in index form.
- 2 **HCF**: take the primes that appear in *every* number, each to its *lowest* power.
- 3 **LCM**: take *every* prime that appears anywhere, each to its *highest* power.

Venn diagram version: shared factors go in the intersection. HCF = product of the intersection. LCM = product of everything.

Which one is the question asking for?

Things happening again *at the same time* (buses, trams, lamps, laps) $\Rightarrow$ **LCM**.
Splitting into the *largest equal groups* with none left over $\Rightarrow$ **HCF**.

Questions 1 and 2

1a $6 = 2 \times 3$, $8 = 2^3$

HCF = 2, LCM = $2^3 \times 3 = 24$

1b 13 and 5 are both prime; nothing shared.

HCF = 1, LCM = $13 \times 5 = 65$

1c $12 = 2^2 \times 3$, $15 = 3 \times 5$

HCF = 3, LCM = $2^2 \times 3 \times 5 = 60$

1d $21 = 3 \times 7$, $35 = 5 \times 7$

HCF = 7, LCM = $3 \times 5 \times 7 = 105$

2a $2^2 \times 3^3$ and $2^3 \times 3^2$

HCF = $2^2 \times 3^2 = 36$

LCM = $2^3 \times 3^3 = 216$

2b $2 \times 3^2 \times 5$ and 2×3^3

HCF = $2 \times 3^2 = 18$

LCM = $2 \times 3^3 \times 5 = 270$

2c $2 \times 3^2 \times 5$ and $3 \times 5 \times 7$

HCF = $3 \times 5 = 15$

LCM = $2 \times 3^2 \times 5 \times 7 = 630$

2d $3^3 \times 5$ and $5^2 \times 7$

HCF = 5

LCM = $3^3 \times 5^2 \times 7 = 4725$

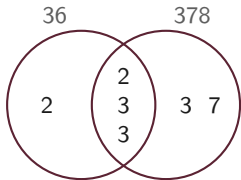
Question 3

3a $36 = 2^2 \times 3^2$

$$378 = 2 \times 3^3 \times 7$$

$$\text{HCF} = 2 \times 3^2 = 18$$

$$\text{LCM} = 2^2 \times 3^3 \times 7 = 756$$



3b $72 = 2^3 \times 3^2$

$$66 = 2 \times 3 \times 11$$

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 2^3 \times 3^2 \times 11 = 792$$

3c $375 = 3 \times 5^3$

$$325 = 5^2 \times 13$$

$$\text{HCF} = 5^2 = 25$$

$$\text{LCM} = 3 \times 5^3 \times 13 = 4875$$

3d $252 = 2^2 \times 3^2 \times 7$

$$2079 = 3^3 \times 7 \times 11$$

$$\text{HCF} = 3^2 \times 7 = 63$$

$$\text{LCM} = 2^2 \times 3^3 \times 7 \times 11 = 8316$$

Questions 4 and 5

4a $60 = 2^2 \times 3 \times 5$

$$18 = 2 \times 3^2$$

$$48 = 2^4 \times 3$$

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 2^4 \times 3^2 \times 5 = 720$$

4b $30 = 2 \times 3 \times 5$, $12 = 2^2 \times 3$,

$$84 = 2^2 \times 3 \times 7$$

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$$

4c $49 = 7^2$, $28 = 2^2 \times 7$, $112 = 2^4 \times 7$

$$\text{HCF} = 7$$

$$\text{LCM} = 2^4 \times 7^2 = 784$$

5 $A = 2^3 \times 5 \times 7^2 \times 11$

$$B = 2^4 \times 7 \times 11$$

$$C = 3 \times 5^2$$

Highest power of each prime:

2^4 (from B), 3 (from C), 5^2 (from C),
 7^2 (from A), 11

$$\text{LCM} = 2^4 \times 3 \times 5^2 \times 7^2 \times 11$$

$$= 16 \times 3 \times 25 \times 49 \times 11$$

$$= 1200 \times 539 = 646\,800$$

Questions 6, 7 and 8

6 Next time together $\Rightarrow$ LCM.

$$9 = 3^2, \quad 12 = 2^2 \times 3$$

$$\text{LCM} = 2^2 \times 3^2 = 36 \text{ minutes}$$

$$9:00 \text{ am} + 36 \text{ min} = \mathbf{9:36 \text{ am}}$$

7 Largest equal team size, nobody left over $\Rightarrow$ HCF.

$$48 = 2^4 \times 3, \quad 90 = 2 \times 3^2 \times 5$$

$$\text{HCF} = 2 \times 3 = \mathbf{6} \text{ pupils per team}$$

(Year 7: 8 teams, Year 8: 15 teams)

8 Next time together $\Rightarrow$ LCM.

$$50 = 2 \times 5^2, \quad 35 = 5 \times 7$$

$$\text{LCM} = 2 \times 5^2 \times 7 = 350 \text{ minutes}$$

$$350 \text{ min} = 5 \text{ h } 50 \text{ min}$$

$$9:30 \text{ am} + 5 \text{ h } 50 \text{ min} = \mathbf{3:20 \text{ pm}}$$

Questions 9 and 10

9 $m = 3^4 \times 5^3$, $n = 3^3 \times 5^2 \times 11$

$$5m = 3^4 \times 5^4$$

$$3n = 3^4 \times 5^2 \times 11$$

$$\text{HCF} = 3^4 \times 5^2 = 81 \times 25 = 2025$$

10 Flashing together again $\Rightarrow$ LCM.

$$20 = 2^2 \times 5$$

$$45 = 3^2 \times 5$$

$$120 = 2^3 \times 3 \times 5$$

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360 \text{ seconds} = 6 \text{ minutes}$$

$$60 \div 6 = 10 \text{ times in one hour}$$

Questions 11 and 12

11a Same number of slices and burgers $\Rightarrow$ LCM.

$$20 = 2^2 \times 5, \quad 12 = 2^2 \times 3$$

$$\text{LCM} = 2^2 \times 3 \times 5 = 60$$

$60 \div 20 = 3$ packets of cheese slices

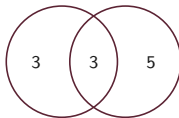
$60 \div 12 = 5$ boxes of burgers

11b 60 slices and 60 burgers, one of each per roll:

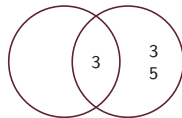
60 bread rolls

12 $\text{HCF} = 3, \quad \text{LCM} = 45 = 3^2 \times 5$

Both numbers contain the 3 from the HCF.
The other factors of the LCM, 3 and 5, are shared out between the two numbers.



9 and 15



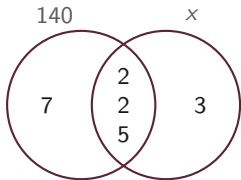
3 and 45

Questions 13 and 14

13 $140 = 2^2 \times 5 \times 7$

$\text{HCF} = 20 = 2^2 \times 5$

$\text{LCM} = 420 = 2^2 \times 3 \times 5 \times 7$



$x = 2^2 \times 5 \times 3 = 60$

Check: $\text{HCF}(140, 60) = 20$,

$\text{LCM}(140, 60) = 420$

14 Together again after 150 s, so

$\text{LCM}(30, t) = 150$.

$30 = 2 \times 3 \times 5$, $150 = 2 \times 3 \times 5^2$

t must contain 5^2 (the 30 only has one 5).

t must divide 150, so it can also include 2, 3, both or neither.

$t = 5^2 = 25$

$t = 2 \times 5^2 = 50$

$t = 3 \times 5^2 = 75$

$t = 2 \times 3 \times 5^2 = 150$ seconds

Questions 15 and 16

15 $16x^6y^3z = 2^4 x^6 y^3 z$
 $24x^4y^6z^5 = 2^3 \times 3 x^4 y^6 z^5$

Lowest power of each shared factor:
 $2^3, x^4, y^3, z$

$$\text{HCF} = 8x^4y^3z$$

16 $42a^3b^5c^2 = 2 \times 3 \times 7 a^3 b^5 c^2$
 $28a^4c^6 = 2^2 \times 7 a^4 c^6$

Highest power of every factor:
 $2^2, 3, 7, a^4, b^5, c^6$

$$\text{LCM} = 2^2 \times 3 \times 7 a^4 b^5 c^6 = 84a^4b^5c^6$$

Questions 17 and 18

17 $\text{LCM}(a, b) = 12 = 2^2 \times 3$
 $\text{LCM}(a, c) = 375 = 3 \times 5^3$

a divides both 12 and 375, so $a = 1$ or 3.

If $a = 3$: b needs 2^2 , so smallest $b = 4$;
 c needs 5^3 , so smallest $c = 125$.

$$\text{LCM}(4, 125) = 2^2 \times 5^3 = 500$$

If $a = 1$: $b = 12$, $c = 375$, $\text{LCM} = 1500$
(larger).

18 $\text{LCM} = 168 = 2^3 \times 3 \times 7$
 $42 = 2 \times 3 \times 7$

42 already supplies 3 and 7, but only one 2.
The 2^3 must come from x .

$$\text{Smallest } x = 2^3 = 8$$

$$\text{Check: } \text{LCM}(8, 42) = 2^3 \times 3 \times 7 = 168$$

Questions 19, 20 and the extension

19 All five numbers are powers of 98.
The LCM must be a multiple of 98^{91} , and 98^{91} is already a multiple of each of the others.

$$\text{LCM} = 98^{91}$$

20 The number of coins must divide exactly by 3, 5 and 7.
 $\text{LCM}(3, 5, 7) = 105$; the next multiple, 210, is over 200.

Answer **B 105**

(35 gold, 21 silver, 30 bronze, 19 copper)

N Each pair's HCF is a different prime. Use the three smallest: 2, 3, 5.

Each number must contain the two primes shared with the other two numbers:

$$Q = 2 \times 3 = 6$$

$$R = 2 \times 5 = 10$$

$$S = 3 \times 5 = 15$$

Check: $\text{HCF}(6, 10) = 2$, $\text{HCF}(6, 15) = 3$,
 $\text{HCF}(10, 15) = 5$

$$Q + R + S = 31$$