

A square matrix A is **singular** when $\det A = 0$. Then A^{-1} **does not exist**.

1 · Algebra: no inverse

For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

If $ad - bc = 0$ we would divide by zero.

Example

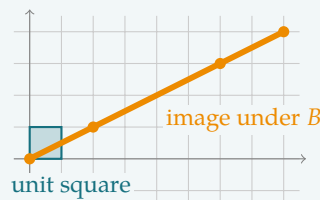
$$B = \begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$$

$$\det B = 2(3) - 6(1) = 0$$

Row 1 is $2 \times$ row 2. The rows are not independent.

2 · Geometry: area collapses

image area = $|\det A| \times \text{object area}$



B sends $(0,0), (1,0), (0,1), (1,1)$ to $(0,0), (2,1), (6,3), (8,4)$: all on $y = \frac{1}{2}x$. Area becomes 0, so the transformation cannot be undone.

3 · Equations: no unique answer

$$\begin{aligned} 2x + 6y &= 10 \\ x + 3y &= 5 \end{aligned} \iff B\mathbf{x} = \mathbf{c}$$

B is singular, so $\mathbf{x} = B^{-1}\mathbf{c}$ is not available.

The two lines are parallel:

- same line $\Rightarrow$ **infinitely many** solutions (here: equation 1 is $2 \times$ equation 2)
- distinct lines $\Rightarrow$ **no** solutions (e.g. right-hand sides 10 and 7)

Eigenvalues: solving $\det(A - \lambda I) = 0$ means finding the λ that make $A - \lambda I$ **singular**.

Careful: a zero entry is not enough. Only $\det A = 0$ makes A singular.